Memorial University of Newfoundland
Department of Physics and Physical Oceanography
Physics 2053 Laboratory

Viscosity of Water

Introduction

When a fluid flows slowly and steadily through a pipe, it may be considered to consist of various layers which move at different velocities relative to each other. Fluid at the centre of a pipe moves at maximum velocity, while the fluid at the edge of the pipe is almost stationary. This is called *Laminar Flow*, illustrated in Figure 1 shows

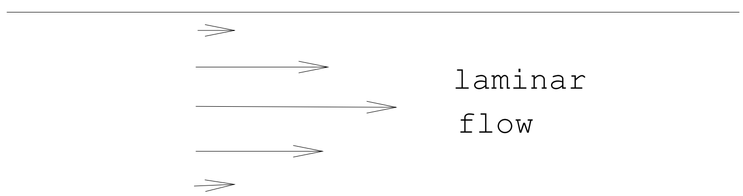

Figure 1: Laminar flow in a pipe

Since each layer moves with a different velocity relative to its neighbour, a frictional force F will exist between layers which depends on the area A of the liquid surface and also on the *rate of shear strain*, or

$$F \propto A \times \text{ rate of shear strain.}$$

where the rate of shear strain is equal to the velocity gradient in the pipe, and hence

$$F = \eta A \times \frac{dv}{dr},$$

where η is the *coefficient of viscosity*, defined as

$$\eta = \frac{\text{shear stress}}{\text{rate of shear strain}}.$$

The units of η are Nsm^{-2} or 'dekapoise'.

Consider a cylinder of fluid of radius r centered on the axis of the pipe of radius a. The surface area of the fluid cylinder (length ℓ) is $2\pi r\ell$. Hence the force exerted by the fluid outside the cylinder on the fluid inside the cylinder is

$$F = -2\pi r\ell\eta\frac{dv}{dr} \tag{1}$$

F opposes the fluid motion inside the cylinder. The '$-$' sign is necessary because v decreases as r increases. For steady flow, a driving force must be applied to counteract the resisting viscous force. If P_1 and P_2 are the pressures at the ends of the fluid element, the net force is given by

$$F = (P_1 - P_2)\pi r^2. \tag{2}$$

Hence we have

$$-2\pi r\ell\eta\frac{dv}{dr} = (P_1 - P_2)\pi r^2 \tag{3}$$

and thus the velocity of a cylindrical shell (radius r) in a cylinder of radius a is given by

$$v(r) = \frac{P_1 - P_2}{4\ell\eta}(a^2 - r^2). \tag{4}$$

The total volume per unit time (i.e. the flow rate or volume flux) through the pipe is obtained by adding up the flow due to all such shells of radius r and thickness dr. i.e.,

$$\frac{dV}{dt} = \int_0^a 2\pi vr\,dr = \frac{\pi a^4(P_1 - P_2)}{8\eta\ell}. \tag{5}$$

This is **Poiseuille's Law** (1835), and holds as long as the flow is laminar; it does not hold for turbulent motion.

Derivation of Poiseuille's Law by Dimensional Analysis

Poiseuille's equation can be derived on the assumption that the volume of liquid flowing through a pipe depends on η, a and the pressure gradient $\Delta p/\ell$. Thus we have

$$\text{volume per second } = k\eta^x a^y \left(\frac{\Delta p}{\ell}\right)^z.$$

Use dimensional arguments to obtain values for x, y and z.

Method

1. A sketch of the experimental setup is shown in Fig. 2. A small electric pump takes water from the tank, and pushes it through the glass tube before being returned to the tank. You may assume that the diameter of the glass tube is 1.2 ± 0.1 mm.

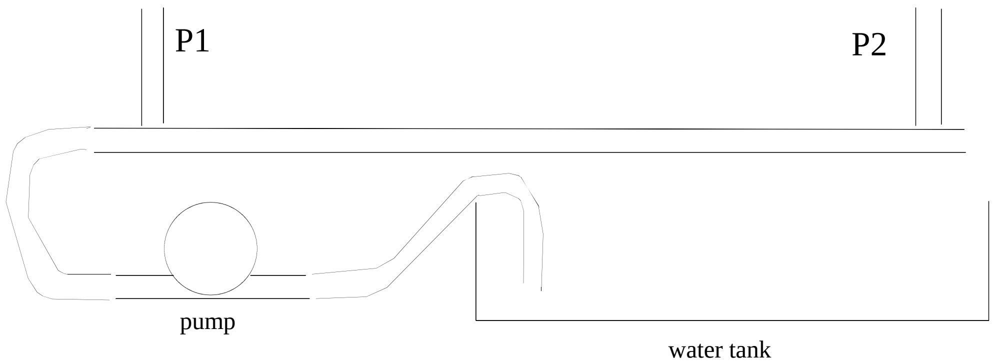

Figure 2: Schematic diagram of the apparatus, including the pump and water reservoir.

2. Start with a low pump speed and record the pressures P_1 and P_2 from the water columns. Determine the flow rate by measuring the amount of water collected in a beaker in one minute.

3. Increase the pump speed slightly, recording the new pressures and flow rate. Repeat, taking as many data points as you can, but do not let the voltage supplied to the pump exceed 12 volts. Plot your data on a suitable graph and determine a value for the viscosity of water (with its associated uncertainty) at room temperature.

4. Estimate the velocity of the water at the centre of the capillary tube when the pressure difference between the ends of the tube is greatest.

Turbulent Flow

Fluid flow can be described in terms of a dimensionless quantity known as the *Reynolds number*.

$$Re = \frac{dv\rho}{\eta} \tag{6}$$

where d is the diameter of the tube and $v = Q/A$ is the velocity of the water in the tube. What is the range of values of Re for your data?

At the *critical velocity*, (denoted by v_c), there is a transition in the flow from laminar to turbulent flow. For fluid flow along a cylindrical pipe, v_c is given by

$$v_c = \frac{R_c \eta}{2a\rho}$$

where ρ is the density of the fluid and R_c is the *critical Reynold's number*, which for most fluids is approximately 2000. Estimate v_c for your configuration.

Can you observe the onset of turbulance?
